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Quality and Price Effects on Technology Adoption

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QUALITY AND PRICE EFFECTS ON TECHNOLOGY ADOPTION

Completed Research Paper

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Abstract

We study the adoption patterns of two competing technologies as well as the effectiveness and optimality of viral pricing strategies. Our model considers two incompatible technologies of differing quality and a market in which valuations are heterogeneous and subject to externalities. We provide partial characterization results about the structure and robustness of equilibria and give conditions under which a technology purveyor can gains in market share. We show that myopic best-response dynamics are monotonic and convergent, and propose two pricing mechanisms using this insight to help a technology seller tip the market in its favor. In particular, we show that non-discriminatory pricing is less costly and just as effective as a discriminatory policy. Finally, we study endogenous pricing using simulations and now find, in contrast to our analytical results with exogenous prices, that a higher quality technology consistently holds a competitive advantage over the lower quality competitor, irrespective of its market share.

Keywords: technology adoption, network externalities, diffusion strategy, network seeding

Introduction

We study the pattern and dynamics of adoption of two technologies of differing qualities, where consumers' valuations are heterogeneous and subject to network effects *(positive externalities)* from the installed base. Such positive externalities occur in a variety of settings: for example, the value of an Internet chat client to a user increases as more of his friends use the same chat client service. Network externalities have an important impact on buyer decisions and can be leveraged to sell a technology more effectively. In this paper, we study the use of simple pricing mechanisms use the network effects in the game to create a cascade of adoption in favor of a higher quality technology.

Throughout, we take the perspective of the higher quality technology seller and study the market through the lens of incomplete information about actual user valuations. As such, we present probabilistic characterizations of equilibrium market shares under exogenously specified price and quality differences between two technologies, which are suggestive of the likelihood of successful market entry of the higher quality technology. We observe, for example, that substantial network effects can often stand in the way of significant initial market penetration, even when the entrant offers a substantial quality improvement.

Even so, our characterization of best response dynamics offers a way for an entrant with the better product to tip the market considerably in its favor. We study two such pricing strategies, which we call "nudging" mechanism, that leverage the monotonicity of best response dynamics and focus on enticing a single non-adopter to make the switch, with the hope that this single switch would create an adoption avalanche. Indeed, our experimental results suggest that the size of such an avalanche is often substantial. Furthermore, under uncertain information, we show that a non-discriminatory nudging mechanism based on a public price subsidy is as effective in growing market share as a discriminatory alternative that "seeds" *(provides a targeted subsidy to)* a sufficient number of non-adopters, all the while costing less. We also address the general problem of profit maximization under uncertainty in our setting for both discriminatory and non-discriminatory pricing regimes. We show that the problem is NP-Hard in general, but observe that the simple nudging approaching can serve as effective heuristics.

Finally, we study the endogenous pricing game between the competing technology sellers. We show, surprisingly, that the seller of a higher quality technology nearly always enjoys a considerable competitive advantage, both through an ability to charge a higher price nearly irrespective of market share, and in obtaining much higher rents from users in the case of market dominance.

Related Work

Modeling the diffusion of new products and technologies has a long tradition in marketing. Fort and Woodlock [1960] propose a product diffusion model in which a fixed fraction of consumers who have not yet bought the product do so at every period. Bass [1969] propose an extension that additionally incorporates word-of-mouth communication between current and potential adopters. A large body of work has since built on these earlier models (see [Mahajan et al., 2000] for a survey). Significantly, both single- and multi-product diffusion models do not explicitly model individual decision-making, focusing instead on aggregate adoption dynamics. While such models make dynamic pricing policies tractable, they fail to explain the endogeneity of outcomes.

The role of network externalities on equilibrium adoption of standards and technologies has received much interest recently. Indeed, an active line of research in economics, mathematical sociology, and marketing is concerned with studying how behaviors, decisions, and trends propagate through a population. These types of diffusion processes naturally suggest a game [Jackson and Yariv, 2007, Morris, 2000], which in turn suggests the use of incentive mechanisms to maximize some specified objective, like profit [Hartline et al., 2008] or influence [Kempe et al., 2003]. Farrell and Saloner [1985] evaluate the impact of an installed base on the transition to a new standard. Cabral [1990] develops a model of individual decision-making in the presence of network externalities and characterizes the aggregate adoption dynamics. Auriol and Benaim [2000] consider a specific model of standard diffusion when users have heterogeneous preferences. A series of recent papers consider adoption dynamics where network externalities are determined by an underlying social network (for example [Sundararajan, 2007, Morris, 2000, Immorlica et al., 2007, Jackson and Yariv, 2007]). Our own approach uses the model of Jin et al. [2008], but while their equilibria are posited as fixed points to a diffusion dynamic, we consider a one-shot game setting that yields different equilibrium structures and adoption dynamics. Furthermore, our focus on pricing mechanisms for a seller is more closely related to the work by Hartline et al. [2008] and Akhlaghpour et al. [2009] on marketing in social networks.

Preliminaries

Suppose there are two competing technologies (1 and 2) with qualities q_1 and q_2 and prices p_1 and p_2 respectively. Let $0 \leq q_k, p_k \leq 1$. We assume throughout that $q_2 > q_1$ and let $\Delta q = q_2 - q_1$ and $\Delta p = p_1 - p_2$. There are N players that are interested in adopting one of these two technologies. Each player i has a parameter $\theta_i \in [0, 1]$ which determines his strength of preference for a higher quality technology. θ_i are distributed i.i.d. according to some distribution $F()$ with density $f()$, which we assume to be strictly positive on $[0, 1]$ and zero everywhere else. Thus, $F()$ is non-atomic and strictly increasing on $[0, 1]$. Let $\bar{F}(\theta) = 1 - F(\theta)$. The utility that player i receives from choosing technology $k \in \{1, 2\}$ is

$$u_i^k = \theta_i q_k + v(x_k) - p_k,$$

where x_k is the proportion of players currently choosing technology k and $v : [0, 1] \to [0, 1]$ is a continuous and strictly increasing function representing network effects on player preferences over the two technologies. Assume that $v(0) = 0$ and $v(1) = 1$. If $x_1 = x$ and $x_2 = 1 - x$, let $\Delta v(x) = v(x) - v(1 - x)$.[1] We now make several observations about $\Delta v(x)$ that will be useful throughout.

Observation 1. $\Delta v(x)$ *is continuous and strictly increasing in* x.

This observation is a direct consequence of the fact that $v(x)$ is continuous and strictly increasing in x.

Observation 2. $\Delta v(0) = -1$, $\Delta v(1/2) = 0$, $\Delta v(1) = 1$.

Observation 3. *If* $v(x) = x$ *or* $v(x) = x^2$, $\Delta v(x) = 2x - 1$.

The previous two are simple algebraic observations that are easy to verify.

Observation 4. $\Delta v(x) = -\Delta v(1 - x)$ *for all* $x \in [0, 1]$.

The final observation follows from definition: $\Delta v(x) = v(x) - v(1 - x)$, while $\Delta v(1 - x) = v(1 - x) - v(x)$.

Observe that the particular realizations of θ_i for all players define a static game between them. One way to look at this game is to assume that θ_i are common knowledge among the players and consider what equilibria can thus arise. Perhaps a more natural alternative way to look at this game is to look at best response dynamics. This is natural because (a) the assumption of common knowledge of what is essentially private information to the players is now unnecessary and (b) it seems more plausible that outcomes of this strategic interaction are a result of some adjustment dynamic rather than one-shot decision-making on the part of the agents. In any case, note that the equilibria of the complete information game are fixed points of best-response dynamics. Looking at the dynamics, however, allows us to additionally consider *which* fixed points arise depending on the initial adoption choices of the players. We now define these equilibria (fixed points) formally.

Let $a_i \in \{1, 2\}$ be a technology choice of player i, which will be subscripted by a time (iteration) t where appropriate.

Definition 1. *A profile of technology choices* a *of all players and corresponding adoption proportions* $x_1 = x$ *and* $x_2 = 1 - x$ *constitute an* equilibrium *if for every* $i \in I$ *with* $u_i^1(x) > u_i^2(x)$, $a_i = 1$ *and for every* $i = 1, \ldots, N$ *with* $u_i^2(x) > u_i^1(x)$, $a_i = 2$.

While we consider iterative best response and its fixed points in the "underlying" complete information game, we take the perspective of a technology seller and so assume that consumer types θ_i are known through a prior distribution $F()$ over these. We view the adoption outcomes and dynamics through the lens of this distribution.

Robustness of Consensus

We begin by considering the two possible consensus equilibria that may arise: one with $x_1 = 1$ ($x_2 = 0$) and one with $x_2 = 1$ ($x_1 = 0$).

[1] While we normally assume that $x_1 + x_2 = 1$, this assumption is not necessary for most of our results and is made primarily for convenience.

Consensus Equilibria

Our first results characterize situations under which each form of consensus is an equilibrium.

Theorem 1. *The probability that $x_1 = 1$ is an equilibrium is $F(\frac{1-\Delta p}{\Delta q})^N$.*

Proof. Suppose $x_1 = x = 1$. It is an equilibrium if every player i prefers technology 1 when $x_1 = 1$, that is, when

$$\theta_i q_1 + v(1) - p_1 \geq \theta_i q_2 + v(0) - p_2,$$

or, equivalently since $v(1) = 1$ and $v(0) = 0$, when

$$\theta_i \leq \frac{1 - \Delta p}{\Delta q} \quad \forall \quad i = 1, \ldots, N.$$

Since $\theta_i \sim F()$ are i.i.d., the result follows. □

From this result it is apparent that if $p_1 \leq p_2$, $x_1 = 1$ is always an equilibrium. On the other hand, if technology 2 holds a significant advantage over 1 in both price and quality, the probability that there is a consensus with $x_1 = 1$ becomes negligible for a large enough N. We now observe that technology 2 enjoys a distinct advantage over 1 due to its superior quality: a consensus on technology 2 is *always* an equilibrium.

Theorem 2. *$x_2 = 1$ is always an equilibrium.*

Proof. Observe that since $q_2 > q_1$, the worst case for technology 2 is when $\theta = 0$. Thus, we need $v(1) - p_2 \geq -p_1$. But since $p_2 \leq 1 = v(1)$ and $p_1 \geq 0$, this is always true. □

Robustness

We now investigate the relative stability or robustness of the two consensus equilibria, by which we mean the likelihood that all players prefer the consensus technology if a fixed random proportion of players switches. A natural interpretation of our robustness results is as tipping points: we in effect discover the points for the two consensus equilibria at which the entire market tips towards these.

Let us begin with consensus on technology 1, a result which may be viewed as a generalization of Theorem 1.

Theorem 3. *All players prefer technology 1 with probability at least $1 - z$ if $x_1 \geq \Delta v^{-1}(\Delta q F^{-1}((1-z)^{1/N}) + \Delta p)$.*

Proof. Let $x_1 = x$, with $x_2 = 1 - x$. To ensure that a single player prefers technology 1, it must be that $\theta_i q_1 + v(x) - p_1 \geq \theta_i q_2 + v(1-x) - p_2$, or, equivalently, $\theta_i \leq \frac{\Delta v(x) - \Delta p}{\Delta q}$. The probability that this is true for all players is

$$F(\frac{\Delta v(x) - \Delta p}{\Delta q})^N.$$

Requiring that this probability is at least $1 - z$ and solving for x yields the desired result. □

Corollary 4. *Suppose that $-\Delta p \geq \Delta q$. Then all players prefer technology 1 with probability 1 if $x_1 \geq 1/2$.*

Proof. Since $1 - z \leq 1$, if $\Delta q \leq -\Delta p$, then $\Delta q F^{-1}((1-z)^{1/N}) + \Delta p \leq 0$. Since $\Delta v^{-1}(0) = 1/2$ and $\Delta v(x)$ is strictly increasing (Observations 2 and 1) the result follows. □

We now turn to the symmetric result for technology 2.

Theorem 5. *All players prefer technology 2 with probability at least $1 - z$ if $x_2 \geq 1 - \Delta v^{-1}(\Delta q F^{-1}(1 - (1 - z)^{1/N}) + \Delta p)$.*

The proof is quite similar to that of the previous theorem and is given in the extended version of the paper.

Corollary 6. *Suppose $\Delta p \geq 0$. Then all players prefer technology 2 with probability 1 if $x_2 \geq 1/2$.*

Proof. If $\Delta p \geq 0$, then $\Delta q F^{-1}(1 - (1-z)^{1/N}) + \Delta p \geq 0$ and the result follows by Observations 1 and 2. □

The robustness results and their corollaries make it very clear that the price difference between the two technologies is critical to the robustness of their respective consensus. Superior quality of technology 2 does not significantly affect robustness of $x_2 = 1$, but it does have a significant affect on the robustness of consensus on technology 1: indeed, if quality of technology 2 is substantially better, it may not require very much to break even exceedingly strong network effects, particularly if, in addition, it is very competitively priced. Nevertheless, breaking the network effects will not necessarily swing the tide entirely towards global consensus on the second technology: if prices are identical, network effects will provide enough buffer for the inferior technology to survive with a substantial market share. Below we provide more concrete results to this effect.

To build some intuition about the robustness of consensus equilibria, consider the following example.

Example 1. *Suppose that $F()$ is a uniform distribution on the unit interval. Let $\Delta v(x) = 2x - 1$, which implies that $\Delta v^{-1}(y) = \frac{y+1}{2}$. For a large enough N, $(1-z)^{1/N} \approx 1$, and we'll take this approximation to be exact below. Then technology 1 is a best response for all players if $x_1 \geq \frac{1}{2} + \frac{\Delta q + \Delta p}{2}$ and technology 2 is a best response for all players if $x_2 \geq \frac{1}{2} - \frac{\Delta p}{2}$.*

In the above example, we clearly see that the relative prices of the two technologies play a crucial role in determining robustness of consensus on either technology. In contrast and somewhat surprisingly, when the pool of users is large, relative quality only plays a role in the robustness of consensus on technology 1, but has no influence on robustness of consensus on the higher quality technology. This fact should give pause to a higher quality technology seller contemplating monopolistic practices even when his lower-quality competitor has negligible market share.

Monotonicity and Structure

Above we focused on analyzing the two consensus equilibria that may arise. We now turn our attention to properties and structure of the *entire set of equilibria*.

Monotonicity

In this section we make several very natural observations about the monotonicity of player preferences, which, while only of limited interest in their own right, will serve as building blocks for other results below. The first lemma states that, given a fixed market share of each technology, if θ is a marginal type (indifferent between two technologies), then a player j with $\theta_j > \theta$ prefers technology 2, whereas a player with $\theta_j < \theta$ prefers technology 1. The second lemma states that increasing the market share of technology 1 will not induce any player who currently prefers technology 1 to switch to technology 2 (the converse is true as well). The relatively straightforward proofs of both these lemmas are provided in the extended version of the paper.

Lemma 7. *Let θ_i be the type of player i, u_i^k the corresponding utility of player i given fixed market shares x_1 and x_2, and suppose that $u_i^1 \geq u_i^2$. Then $\theta_j < \theta_i$ for some player j implies that $u_j^1 > u_j^2$. Similarly, if $u_i^1 \leq u_i^2$, then $\theta_j > \theta_i$ for some player j implies $u_j^1 < u_j^2$.*

Lemma 8. *Fix a player i with type θ_i and suppose that $x_1 \leq x_1'$ and $u_i^1(x_1) \geq u_i^2(x_1)$. Then $u_i^1(x_1') \geq u_i^2(x_1')$. Conversely, if $x_1 \geq x_1'$ and $u_i^1(x_1) \leq u_i^2(x_1)$, then $u_i^1(x_1') \leq u_i^2(x_1')$.*

The first useful result that follows directly from these lemmas is that every equilibrium corresponds to some $\bar{\theta}$ which separates all player types into those who prefer technology 1 and those who prefer technology 2. We show this formally in the following section.

Structure of Equilibria

We being our exploration of the structure of equilibria by showing that each equilibrium can be described by a threshold $\bar{\theta}$ with the property that all players with $\theta_i \leq \bar{\theta}$ choose (and prefer) technology 1 and all players with $\theta_i > \bar{\theta}$ choose (and prefer) technology 2. Without loss of generality and to simplify the analysis, we assume from now on that all indifferent players select technology 1.

Theorem 9. *Let a be an equilibrium profile with corresponding $x_1 = x$ and $x_2 = 1 - x$. Then there exists $\bar{\theta}$ such that $\theta_i \leq \bar{\theta}$ if and only if $a_i = 1$.*

Proof. Let θ be such that given x, the player with preference θ is indifferent between the two technologies. Then by Lemma 7, any i with $\theta_i < \theta$ strictly prefers technology 1, whereas any i with $\theta_i > \theta$ strictly prefers technology 2. Letting $\bar{\theta} = \theta$ yields the result. □

In Section we generalize this result and show that this strict ordering and the existence of such a threshold is true for any best response profile a. Since in general there can be a continuum of $\bar{\theta}$ that can serve as the threshold, it will be convenient to let $\bar{\theta}$ be the greatest type θ_i which prefers technology 1. This will indeed be our convention.

We can make several straightforward observations about the structure of the set of equilibria $\bar{\theta}$ when the parameters of the players' utility functions change. First, note that if either Δp or Δq increases, every player i with $\theta_i > \bar{\theta}$ for a fixed equilibrium $\bar{\theta}$ will (weakly) prefer technology 2. Hence, the set of equilibria will (weakly) shift up. This observation roughly corresponds to the greater diffusion property of Jackson and Yariv [2007]. To support a more formal discussion below, we now define precisely our notion of greater diffusion (or "shifting up") of a (sub)set of equilibria.

Definition 2. *Let $\Theta(r)$ be a (sub)set of equilibria under problem parametrization r. We say that $\Theta(r)$* shifts up *under another parametrization r' if for every $\bar{\theta} \in \Theta(r)$, every player i with $\theta_i > \bar{\theta}$ prefers technology 2 under r'. Similarly, $\Theta(r)$* shifts down *under r' if for every $\bar{\theta} \in \Theta(r)$, every player i with $\theta_i < \bar{\theta}$ weakly prefers technology 1 under r'.*

The parametrization r referred to in the above definition could be, for example, a specific choice of Δq or a choice of the network effects function $v(x)$.

We next consider how the shape of $v(x)$—that is, the impact of a change in market shares on network externalities—affects the set of equilibria. What we find, somewhat surprisingly, is that it is not the convexity properties of $v(x)$, but the shape of its derivative that are pertinent.

Lemma 10. *Suppose that $v_1(x)$ and $v_2(x)$ are twice continuously differentiable and suppose that $\Delta v_1''(x) \geq \Delta v_2''(x)$ for $x \in [0, 1/2]$. Then $\Delta v_1(x) \geq \Delta v_2(x)$ for all $x \in [0, 1]$.*

The proof is rather involved and is given in the extended version.

Lemma 11. *Suppose $\Delta v_1(x) \geq \Delta v_2(x)$ on $x \in X \subset [0, 1]$ and let $\bar{\theta}$ be an equilibrium with $\bar{x} \in X$ the corresponding market share of technology 1 under $v_2(x)$ (corresponding to $\Delta v_2(x)$). If $\bar{x}$ remains fixed, then every i with $\theta_i > \bar{\theta}$ will prefer technology 2 under $\Delta v_1(x)$.*

Proof. Since $\bar{\theta}$ is an equilibrium under $\Delta v_1(x)$ and since player i with $\theta_i > \bar{\theta}$ prefers technology 2 under this equilibrium, $\theta_i q_2 + v_2(\bar{x}) - p_2 \geq \theta_i q_1 + v_2(1 - \bar{x}) - p_1$, or, equivalently, $\theta_i \Delta q + \Delta v_2(\bar{x}) - \Delta p \geq 0$. Since $\Delta v_1(x) \geq \Delta v_2(x)$, it is immediate that player i will prefer technology 2 under $\Delta v_1(x)$. □

The following theorem uses these lemmas to characterize how the shape of $v(x)$ can affect the direction of equilibrium shift.

Theorem 12. *Assume that $v_1(x)$ and $v_2(x)$ are three times continuously differentiable. Suppose that $v_1'(x)$ is concave and $v_2'(x)$ is convex and let Θ be the set of all equilibria under $v_1(x)$. Then Θ (weakly) shifts up under $v_2(x)$.*

Proof. Let $x \in [0, 1/2]$. Then for concave $v_1'(x)$, $\Delta v_1''(x) \geq 0$, while for convex $v_2'(x)$, $\Delta v_2''(x) \leq 0$. To see this, note that $\Delta v''(x) = v''(x) - v''(1 - x)$ for any $\Delta v(x)$. If $v'(x)$ is concave, $v''(x)$ is decreasing in x. Thus, since $x \leq 1/2$, $x \leq 1 - x$ and, consequently, $v''(x) \geq v''(1 - x)$, which implies that $\Delta v''(x) \geq 0$. The reverse is true if $v'(x)$ is convex.

Now, since $\Delta v_1''(x) \geq 0$ and $\Delta v_2''(x) \leq 0$, we have that $\Delta v_1''(x) \geq \Delta v_2''(x)$ for all $x \in [0, 1/2]$. Thus, by Lemma 10, $\Delta v_1(x) \geq \Delta v_2(x)$ for all $x \in [0, 1]$. By Lemma 11, then, for any equilibrium $\bar{\theta}$ players with $\theta_i > \bar{\theta}$ will prefer technology 2 under $v_1(x)$ if they did under $v_2(x)$. Consequently, the set of equilibria shifts up if $v_2(x)$ is replaced by $v_1(x)$. □

The upshot of Theorem 12 is that the set of equilibria is "higher" in the sense of Definition 2 when $v'(x)$ is convex as compared to a $v(x)$ with a concave derivative. Thus, interestingly, convexity of the derivative of the network effects function favors the higher quality technology.

Entrant Adoption Dynamics

We now attempt to assess how large the initial market share of technology 1 can be to still allow a higher quality entrant to make significant inroads.

Distribution-Independent Results

To begin, we consider what we can say with *complete certainty*, that is, results which we can produce that are independent of the specific distribution of player types $F()$. The key question that we raise in this vein is how small does $x = x_1$ have to be (assuming $x_2 = 1 - x_1$) in order for all agents to (strictly) prefer technology 2. Since type is a multiplier of quality, any distribution-independent result in this direction must hold for the least type, that is, for $\theta = 0$. Hence, all agents will prefer technology 2 if $v(x) - p_1 < v(1-x) - p_2$ or, equivalently, if $\Delta v(x) = v(x) - v(1-x) < \Delta p$. Let x^* be a solution to

$$\Delta v(x) = \Delta p. \tag{1}$$

Since $\Delta v(x)$ is continuous and strictly increasing in x and $\Delta v(0) = -1$ and $\Delta v(1) = 1$ (Observations 1 and 2), there exists a unique solution x^* to this for every $-1 \leq \Delta p \leq 1$ and, additionally, the requisite inequality holds for any $x < x^*$. Furthermore, it is clear that if $\Delta p < 0$, $x^* < 1/2$, and, conversely, $\Delta p > 0$ implies $x^* > 1/2$. We now show that x^* is increasing in Δp.

Theorem 13. *Let x^* solve (1). Then $\frac{dx^*(\Delta p)}{d(\Delta p)} > 0$.*

Proof. Define $f(x, \Delta p) = \Delta v(x) - \Delta p$. Then $f(x^*, \Delta p) = 0$ and

$$\left.\frac{\partial f}{\partial x}\right|_{x^*} = \Delta v'(x)|_{x^*} > 0$$

since $\Delta v(x)$ is strictly increasing in x. Thus, by the Implicit Function Theorem,

$$\frac{dx^*(\Delta p)}{d(\Delta p)} = -\left.\frac{\frac{\partial f}{\partial(\Delta p)}}{\frac{\partial f}{\partial x}}\right|_{x^*} = \frac{1}{\Delta v'(x^*)} > 0.$$

□

For concreteness, consider the following example.

Example 2. *Suppose $v(x) = x$. Then the condition 1 implies that $x^* = \frac{1+\Delta p}{2}$, and, consequently, $\frac{dx^*}{d(\Delta p)} = 1/2$.*

The example suggests that while network effects can cause a lower-quality technology to stick, lowering the price has a significant constant impact when network effects are linear, and predatory pricing or subsidies may ultimately put the higher quality technology in a position for a successful entry.

Distribution-Dependent Results

In this section we consider the above question with a probabilistic nuance. Specifically, we are concerned with the maximal size of x_1 such that at least some specified proportion of all players strictly prefer technology 2 with probability at least $1 - z$. From the business strategy or policy perspective, this would tells us, for example, how much we need to seed the market with a new superior technology before network effects begin to carry it.

As before, let $x_1 = x$ and $x_2 = 1 - x$. Recall from Corollary 6 above that if $\Delta p > 0$, that is, if the superior quality product also has a price advantage, controlling at least 1/2 of the market is sufficient to ensure that all players will prefer technology 2 after two best response iterations. Hence, if it is likely for some $x_1 > 1/2$ that at least 1/2 of the players prefer technology 2, then superior technology will ultimately (and quite quickly) come to dominate.

Let k be the number of players we want to prefer technology 2. The condition we need to hold is that

$$\Pr\{\theta_{(k)} q_2 + v(1-x) - p_2 > \theta_{(k)} q_1 + v(x) - p_1\} \geq 1 - p,$$

where $\theta_{(k)}$ is the kth order statistic. This can be obtained if

$$\Pr\left\{\theta_{(k)} > \frac{\Delta v(x) - \Delta p}{\Delta q}\right\} \geq \Pr\left\{\theta > \frac{\Delta v(x) - \Delta p}{\Delta q}\right\}^k \geq 1 - z.$$

Hence, we have the following result, restated in terms of proportion y of players adopting technology 2.

Theorem 14. *Suppose that x^* solves*

$$F\left(\frac{\Delta v(x) - \Delta p}{\Delta q}\right) = 1 - (1 - z)^{1/(yN)}.$$

Then whenever $x_1 \leq x^*$, with probability at least $1 - z$ at least a proportion y of N players prefer technology 2.

As an example, the following bound is obtained when $F()$ is a uniform distribution:

$$x^* \leq \frac{\Delta q}{2}(1 - (1 - p)^{1/(yN)}) + \frac{1 + \Delta p}{2}. \tag{2}$$

The following example illustrates the bound at work.

Example 3. *Let $N = 10$, $y = 1/2$, $p = 0.7$, $\Delta p = 0$, and $\Delta q = 1/2$. Then $x^* \leq 0.55$.*

The example demonstrates that even when the quality of the entrant's technology is far superior and prices are identical, significant network effects are extremely difficult to overcome, and predatory pricing or subsidies may actually be necessary to achieve substantial market penetration.

Best-Response Dynamics

In this section we demonstrate a key monotonicity property of best response dynamics in our setting.[2] The implication of this property is that best response *always* converges.

To begin, let us introduce some notation. Let n_t denote the number of players in iteration t that adopt technology 1. Let $x_t = \frac{n_t}{N}$ be the fraction of all players adopting technology 1 and let $1 - x_t$ be the fraction of players adopting technology 2. The dynamics will be characterized by the change in the numbers (and, consequently, proportions) of players adopting technology 1 as iterations progress. Let $a_{i,t}$ be the adoption strategy of player i at time t. Thus, $a_{i,t} = 1$ means that player i chooses technology 1 in time t. Hence, $n_{t+1} = n_t + \Delta n_t$, where

$$\Delta n_t = |\{i : a_{i,t+1} = 1, a_{i,t} = 2\}| - |\{i : a_{i,t+1} = 2, a_{i,t} = 1\}|.$$

Definition 3. *Let a_t be a vector of player strategies at time t, n_t the corresponding number of adopters of technology 1, x_t the corresponding proportion of technology 1 adopters. a_{t+1} is a* best response *to a_t when for all i,*

$$\theta_i q_1 + v(x_t) - p_1 > \theta_i q_2 + v(1 - x_t) - p_2 \Rightarrow a_{i,t+1} = 1$$

and

$$\theta_i q_1 + v(x_t) - p_1 < \theta_i q_2 + v(1 - x_t) - p_2 \Rightarrow a_{i,t+1} = 0.$$

Lemma 15. *Suppose that at time t players make some arbitrary choices a_t which result in some x_t. Let a_{t+1} be a best response to a_t. Then there is $\bar{\theta}_{t+1}$ such that $\forall \theta_i \leq \bar{\theta}_{t+1}, a_{i,t+1} = 1$ and $\forall \theta_i > \bar{\theta}_{t+1}, a_{i,t+1} = 2$.*

Since the proof is quite similar to that of Theorem 9, we defer it to the extended version of the paper.

By Lemma 15, we can assume without loss of generality that there exists such $\bar{\theta}_0$ which separates the players into those who prefer technology 1 and those who prefer 2. We will therefore assume that for every t there is some player $\theta_j = \bar{\theta}_t$ with $\bar{\theta}_t$ as in Lemma 15.

Lemma 16. *Let $\theta_j = \bar{\theta}_t$ and suppose that $\bar{\theta}_t$ is not an equilibrium. Then either (a) $u_j^1 < u_j^2$ or (b) $u_{j+1}^1 > u_{j+1}^2$.*

[2] Since our setup yields a supermodular game, convergence would be guaranteed if we additionally knew that equilibrium is unique [Milgrom and Roberts, 1990]. In our setting, however, uniqueness of equilibrium is an exception rather than the rule.

Proof. Suppose that neither (a) nor (b) hold, but $\bar{\theta}_t$ is not an equilibrium. That means that $u_j^1 \geq u_j^2$ and $u_{j+1}^2 \geq u_{j+1}^1$. By Lemma 7, this implies that for all $j' \leq j$, $u_{j'}^1 \geq u_{j'}^2$ and for all $j' > j$, $u_{j'}^2 \geq u_{j'}^1$, which means that $\bar{\theta}_t$ is an equilibrium, a contradiction. □

We now define a best (or better) response dynamics.

Definition 4. *A sequence* $\{a_t\}_t$ *is a* best response (BR) dynamics *if* a_{t+1} *is a best response to* a_t *for all* t.

Definition 5. $\{x_t\}_t$ *is* monotone *if* $x_1 \leq x_0 \Rightarrow x_{t+1} \leq x_t \ \forall t$ *and* $x_1 \geq x_0 \Rightarrow x_{t+1} \geq x_t \ \forall t$.

The following result establishes that best response dynamics is monotone in the above sense, and from this it will follow directly that best response dynamics converges to an equilibrium from any starting point.

Theorem 17. *If* $\{x_t\}_t$ *is generated by best response dynamics, then it is monotone for* $t \geq 1$*. Furthermore,* $x_{t+1} = x_t$ *if and only if* $\bar{\theta}_t$ *is an equilibrium.*

Proof. Note that by Lemma 15, for all $t > 0$, there is $\bar{\theta}_t$ which separates player types linearly based on preferences. Thus, let $t \geq 1$. Now, assume that condition (a) in Lemma 16 holds at time t. Then $\Delta n_t < 0$ since by Lemma 7 we are guaranteed that all players adopting 2 at time t will still prefer it at time $t+1$, and (a) means that, additionally, the marginal type prefers it also. If $\Delta n_t < 0$, then clearly $x_{t+1} < x_t$. By Lemma 15, there again exists $\bar{\theta}_{t+1}$ which separates player types linearly based on their preferences. Then, if x_{t+1} (and corresponding $\bar{\theta}_{t+1}$) is an equilibrium, it remains constant forever after. Suppose x_{t+1} is not an equilibrium. By Lemma 8 and Lemma 16 it follows that condition (a) obtains again (i.e., it cannot be that (b) holds by Lemma 8), and, hence (by the same argument) $\Delta n_{t+1} < 0$. By induction, then, $\Delta n_{t'} \leq 0$ for all $t' \geq t$, and the sequence is stationary if and only if the equilibrium is reached. If we assume that condition (b) holds in Lemma 16, the result can be proved by a symmetric argument. □

Corollary 18. *Best response dynamics converges to some equilibrium from any starting point.*

Proof. This follows from the fact that best response dynamics yields a monotone sequence $\{x_t\}_t$ which is strictly monotone unless x_t is an equilibrium for some t, and the fact that $0 \leq x_t \leq 1$. □

As we will see below, monotonicity of best response dynamics allows us to use very simple techniques to affect substantial changes in the market shares of the two technologies and, thus, overcome what may otherwise seem like impenetrable network effects.

Better Adoption by "Nudging"

The monotonicity of best response dynamics is a rather powerful property, and an immediate question is how can one take advantage of it in effecting adoption of a socially preferred technology (in our case, technology 2).[3] In this section we introduce the idea of "nudging". The metaphor comes from the observation that we need only to get one more player to adopt the entrant technology in order to create a kind of avalanche in adoption until a new equilibrium is reached. Furthermore, by monotonicity of best response dynamics this new equilibrium would have more adopters of the entrant technology.

To begin, suppose that the market shares of competing technologies are in equilibrium with x the proportion of players adopting technology 1. Let i be the number of players who adopt technology 2 under this equilibrium; hence, $x = 1 - i/N$.

Nudging via Seeding

Our first mechanism for effecting a change is via seeding a random sample of players who are technology 1 adopters under the current equilibrium. We would like to know how many players we have to seed in order to have an additional voluntary adoption which (the hope is) will start an adoption avalanche.

Since we know that any additional player who we would like to adopt technology 2 prefers technology 1 under x, we have that $\theta q_1 + v(x) - p_1 \geq \theta q_2 + v(1-x) - p_2$, or, equivalently, $\theta \leq \frac{\Delta v(x) - \Delta p}{\Delta q}$. Now, define

$$f(x) = F\left(\frac{\Delta v(x) - \Delta p}{\Delta q}\right). \tag{3}$$

[3] Alternatively, how can the owner of technology 2 (say, in a case of entry with technology 1 as the incumbent) create an adoption avalanche.

We would like to select k players to seed such that, for $x' = 1 - \frac{i+k}{N}$, $\theta q_1 + v(x') - p_1 < \theta q_2 + v(1-x') - p_2$. Rearranging, we need $\theta > \frac{\Delta v(x') - \Delta p}{\Delta q}$ to incentivize a switch to technology 2. Since we don't know the specific types of players but only their distribution, we would like this condition to be likely, that is, we want

$$\Pr\left\{\theta > \frac{\Delta v(x') - \Delta p}{\Delta q} | \theta \leq \frac{\Delta v(x) - \Delta p}{\Delta q}\right\} \geq 1 - z$$

for some choice of (small) z.

$$\begin{aligned}\Pr\{\theta > \frac{\Delta v(x') - \Delta p}{\Delta q} | \theta \leq \frac{\Delta v(x) - \Delta p}{\Delta q}\} = \\ &= \frac{\Pr\{\frac{\Delta v(x') - \Delta p}{\Delta q} < \theta \leq \frac{\Delta v(x) - \Delta p}{\Delta q}\}}{\Pr\{\theta \leq \frac{\Delta v(x) - \Delta p}{\Delta q}\}} \\ &= \frac{f(x) - F(\frac{\Delta v(x') - \Delta p}{\Delta q})}{f(x)} \geq 1 - z.\end{aligned}$$

Solving for k we obtain the following result.

Theorem 19. *Given an equilibrium x, one or more of current technology 1 adopters will prefer technology 2 with probability at least $1 - z$ if k or more technology 1 adopters are randomly seeded, where*

$$k = N - i - \Delta v^{-1}(\Delta q F^{-1}(z f(x)) + \Delta p). \tag{4}$$

To determine the *proportion* of technology 1 adopters that we need to seed, rewrite Equation 4 as

$$x_{seed} = \frac{k}{N - i} = 1 - \frac{\Delta v^{-1}(\Delta q F^{-1}(z f(x)) + \Delta p)}{N - i}. \tag{5}$$

It would be of some value to know that x_{seed} decreases as x decreases (that is, as technology 2 becomes more prevalent). Note that this is not necessarily the case: while the network effects increasingly favor technology 2 as x_2 rises, we also need to persuade agents with a lower θ to switch. Additionally, we are computing the proportion of a decreasing population, so even if the actual number of players we need to seed decreases, the proportion may not. As the following theorem shows, x_{seed} does, indeed, decrease with decreasing dominance of the first technology (i.e., decreasing x) when $F()$ is a uniform distribution over the unit interval and $v(x) = x$.[4]

Theorem 20. *Suppose that $F(\theta) = \theta$ and $\Delta v(x) = 2x - 1$. Then $\frac{dx_{seed}}{dx} > 0$.*

Proof. If $\Delta v(x) = 2x - 1$, $\Delta v^{-1}(y) = \frac{y+1}{2}$. Similarly, if $F(\theta) = \theta$, $F^{-1}(\theta) = \theta$ and

$$f(x) = \frac{\Delta v(x) - \Delta p}{\Delta q} = \frac{2x - 1 - \Delta p}{\Delta q}.$$

Plugging into Equation 5 we have

$$\begin{aligned} x_{seed} &= 1 - \frac{\Delta q z f(x) + \Delta p + 1}{2(N - i)} = \\ &= 1 - \frac{z(2x - 1 - \Delta p) + \Delta p + 1}{2Nx} = \\ &= 1 - \frac{2zx + (1 - z)(1 + \Delta p)}{2Nx} \end{aligned}$$

Differentiating x_{seed} with respect to x yields

$$\begin{aligned} \frac{dx_{seed}}{dx} &= -\left[-\frac{2zx + (1 - z)(1 + \Delta p)}{2Nx^2} - \frac{2z}{2Nx}\right] \\ &= \frac{(1 - z)(1 + \Delta p)}{2Nx^2} > 0. \end{aligned}$$

Incidentally, we may observe that x_{seed} is decreasing at an increasing rate as x falls, which is due to the increasing influence of network effects on the dynamics. □

[4] Actually, a slightly weaker condition suffices.

The Cost of Nudging via Seeding

While "nudging" seems to be an appealing way to influence adoption of some technology, namely the higher quality technology as we have formally studied it, it does not come for free: there needs to be some mechanism by which the players who are selected to be seeded are actually led to prefer technology 2. The simplest such mechanism is by offering them targeted subsidies for adopting the desired technology. The question is how much do we expect to pay?

To answer this, suppose that upon doing the above analysis we arrived at a number k of players that we want to seed and we have randomly chosen these from the population of players who are currently technology 1 adopters. There are two possibilities that we may consider. The first possibility is that we offer the same payment to all players and, thus, need the payment to be sufficient to incentivize all of those selected to switch to technology 2, no matter what type. In that case, the condition we need to hold is $v(x) - p_1 \leq v(1-x) - p_2 + b$, or, equivalently, the smallest incentive b that suffices is $b = \Delta v(x) - \Delta p$ and the total payout would then be $kb = k(\Delta v(x) - \Delta p)$.

In practice, we could, perhaps, do somewhat better than that. Suppose that instead of offering a fixed incentive to every one of the k chosen players, we offer each of them an increasing sequence of payments until they finally accept the incentive and switch. In this case (at least if the players are entirely myopic and do not game such a system), we need a weaker condition to hold for each player i:

$$\theta_i q_1 + v(x) - p_1 \leq \theta_i q_2 + v(1-x) - p_2 + b,$$

which gives us a minimal subsidy to induce a switch of a player with type θ of

$$b_{min}(\theta) = \Delta v(x) - \Delta p - \theta \Delta q.$$

The ex ante expected payment to any of these k players is then

$$b_{ave} = E[b_{min}(\theta)] = \Delta v(x) - \Delta p - E[\theta | \theta \leq f(x)] \Delta q, \tag{6}$$

and the expected total payment is just kb_{ave}. In the special case of $F(\theta)$ uniform,

$$E[\theta | \theta \leq f(x)] = \int_0^{f(x)} \theta \frac{f(\theta)}{F(f(x))} = 0.5 f(x). \tag{7}$$

Nudging via Posted Prices

Above we presented a technique for incentivizing one player to switch from his current equilibrium preference of technology 1 to adopt technology 2 by seeding some proportion of technology 1 adopters. A natural alternative to the seeding approach is to simply decrease the price of the second technology sufficiently to induce the player on the "boundary" to switch. The question is then what the new price needs to be in order to incentivize the desired switch with desired probability. Let us say that p_2 is the initial price of technology 2, whereas p_2' is the price we would like to set and let $\Delta p' = p_1 - p_2'$.

Theorem 21. *Given an equilibrium x, one or more of current technology 1 adopters will prefer technology 2 with probability at least $1 - z$ if the price of technology 2 is lowered to*

$$p_2' \leq p_1 - \Delta v(x) + \Delta q F^{-1}(z f(x)). \tag{8}$$

Proof. Much of the analysis is analogous to that for nudging via seeding. By similar arguments as above, we obtain the condition that

$$\frac{f(x) - F(\frac{\Delta v(x) - \Delta p'}{\Delta q})}{f(x)} \geq 1 - z$$

which yields $\Delta v(x) - \Delta p' \leq \Delta q F^{-1}(z(f(x)))$. Solving for p_2' we obtain the desired result. □

Now, if we are interested in determining how much each player who ultimately adopts technology 2 is, in effect, "paid" (i.e., how much profit we lose per player by lowering the price), we can compute $p_2 - p_2'$:

$$b_p = p_2 - p_2' = (\Delta v(x) - \Delta p) - \Delta q F^{-1}(z f(x)).$$

Observe that this per-player cost is strictly less than the worst-case cost of seeding each player. The comparison with *expected* per-player seeding cost is less clear in general: $b_p \geq b_{ave}$ if and only if $F^{-1}(z f(x)) \leq E[\theta | \theta \leq f(x)]$. We can, however, obtain a rather crisp result in the case when $F(\theta)$ is uniform on the unit interval.

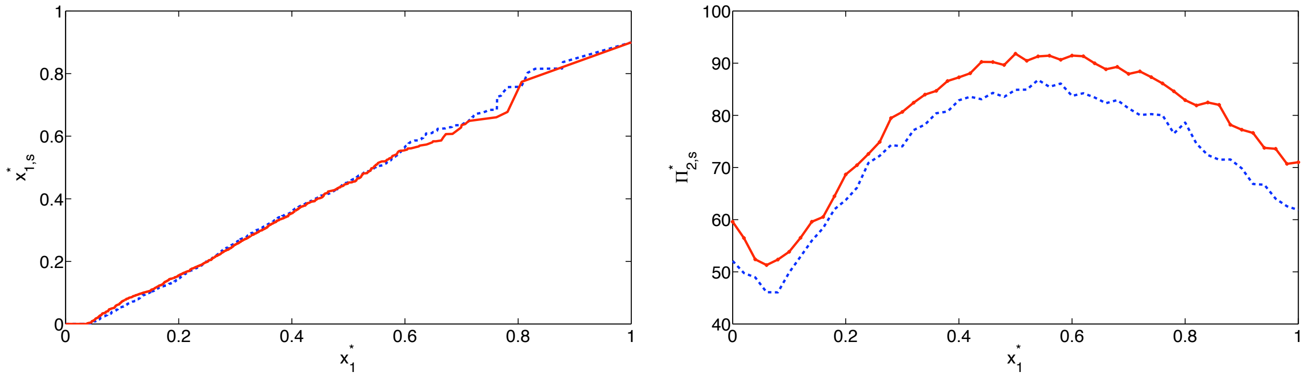

Figure 1: The lefthand plot shows the new equilibrium market share of technology 1,x^*_1, as a function of a starting equilibrium x^o_1 for nudging by posted price discount *(red solid line)* and by seeding *(blue dashed line)*. The right-hand plot shows the profit increase achieved by technology firm 2 in the new equilibrium as a function of a starting equilibrium x^o_1 for nudging by posted price discount *(red solid line)* and by seeding *(blue dashed line)*. Parameters: $N = 1000, z = 0.5, \Delta p = 0.1, \Delta q = 0.1$.

Theorem 22. *Suppose that $F(\theta)$ is uniform on the unit interval. Then $b_p \geq b_{ave}$ if and only if $z \leq 0.5$.*

Proof. If $F(\theta)$ is uniform, $F^{-1}(zf(x)) = zf(x)$. By (7), $E[\theta|\theta \leq f(x)] = 0.5f(x)$. Hence, $b_p \geq b_{ave}$ if and only if $z \leq 0.5$. $\square$

We do want to note that the direct comparison is not entirely fair unless $z = 1/2$, since in the case of seeding with targeted incentives we are considering expected payment, the results in this section are for a target probability.

Indeed, the key difference between the targeted approach and the one based on pricing is the difference in the number of players to whom the effective subsidy is made. We can actually observe that the pricing approach carries a marked advantage if we note that the subsidies need not be made at all to the players who are already technology 2 adopters, but can, rather, target only those who are not yet in the network! This would, indeed, be rather analogous to the lower prices offered by the wireless telephone companies to the customers who are currently under a different carrier.

Profit Maximization by Nudging

As discussed earlier, "nudging" mechanisms, whether by a posted price discount or by seeding (discriminatorily), can be used to increase market share, but they can also be leveraged to maximize other seller objectives. In this section we address the algorithmic complexity of profit maximization by nudging. In keeping with earlier sections, we solve the problem from the perspective of the higher quality technology, 2. We formalize the profit maximization (PM) problem as follows under discriminatory and non-discriminatory regimes, in both cases with the assumption that cost is fixed.

Definition 6 (Non-Discriminatory PM Nudging)**.** *Given a fixed competing price p_1, an initial price p^o_2 for technology 2, quality difference $\Delta q > 0$, and initial market share of technology 2 x^o_2, our goal is to find a sequence $p_1, \ldots, p_K$ of $K \in \mathbb{N}^+$ of public prices such that the final equilibrium market share for technology 2, x^*_2 (at price p_K), maximizes the expected profit to firm 2. At the start of each time step $k \leq K$, each consumer decides whether or not to buy the item at price p_{k-1}, and consumers' decisions are not affected by the actions of other buyers in the same time step.*

Put another way, the above problem amounts to making incremental adjustments to firm 2's public offering price in a way that maximizes its profit in the final state. Note that we have not placed any constraints on the number of price adjustments. Indeed, if profit is maximized for $x^*_2 > x^o_2$ then it is easy to show that that $p_k < p_{k-1} < \cdots \leq p^o_2$ for all $k \leq K$ and the firm 2 will wait to adjust its posted price until no further consumers are willing to adopt technology 2. But this is generally not true.

Definition 7 (Discriminatory PM Nudging)**.** *In the discriminatory profit maximization setting, our goal is now to find a set of targeted discounted prices $\{p^i_k\}$ for all non-adopters $i \in x^o_1$, applied in the sequence $k \leq Nx^o_1$ to maximize profit given the probability distribution $F(\theta)$ and an initial equilibrium market share x^o_2.*

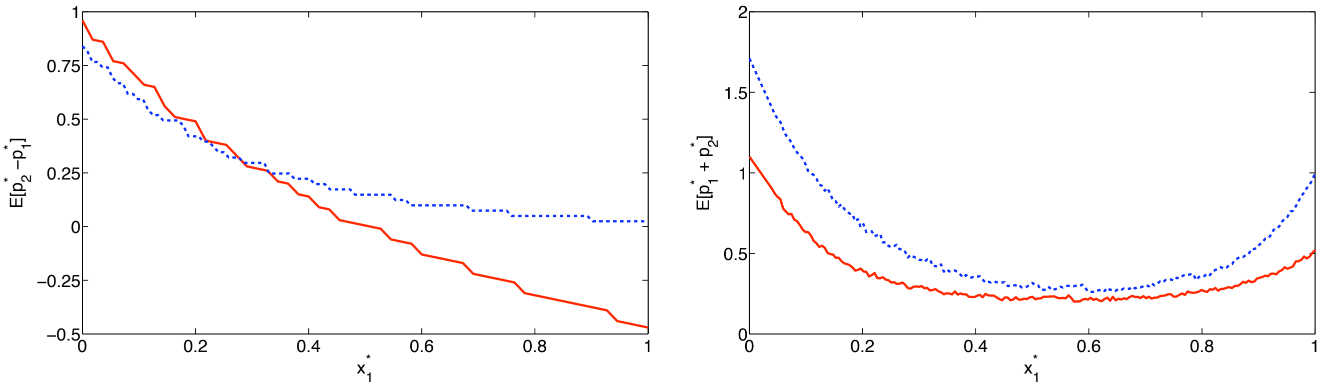

Figure 2: The plots show $E[p_2 - p_1]$ and $E[p_1 + p_2]$ as a function of the equilibrium market share of technology 1 when $N = 1000$ for $\Delta q = 0.1$ *(blue dashed line)* and $\Delta q = 0.05$ *(red solid line)*.

Hardness

We now contrast the algorithmic problem of solving the discriminatory and non-discriminatory PM nudging problems. We show that under complete information, when consumer types are known, nudging optimally by posted discount, i.e. without discriminatory pricing, is NP-Hard, whereas seeding, a discriminatory policy, is easily solvable by a greedy approach. Under uncertainty, nudging by seeding also becomes NP-Hard.[5]

Computing an optimal posted price is hard even under complete information about player preferences. The proof uses a reduction from the independent set problem.[6]

Theorem 23. *Profit maximization by non-discriminatory nudging is NP-Hard even with complete information about consumer types.*

Under the same conditions of complete information, the situation is markedly different, however, if the seller is allowed to price-discriminate: in such a case, the problem becomes easy. To see this, consider the following scheme: rank players in order of type θ_i, and charge each player the price p_2^i that just incentivizes them to adopt technology 2 given x_2^o. This scheme will result in universal adoption and changing any of the prices p_2^i will result either in profit loss or in the loss of adopter to technology 1.[7]

If we now return to the incomplete information setting, profit maximization becomes hard under both discriminatory and non-discriminatory pricing, which can be shown by a reduction to the maximum feedback arc set problem.

Theorem 24. *Profit maximization is NP-Hard under incomplete information about consumer types for both discriminatory and non-discriminatory nudging mechanisms.*

Whereas complete information may offer considerable algorithmic advantage to certain pricing schemes to increase market share in a profit-optimal fashion, no such respite is available under incomplete information. Under uncertainty about buyers' preferences, even against a non-responsive competitor, i.e. when firm 1 does not respond to firm 2's pricing strategy, firm 2 invariably faces a hard problem.

Comparing Nudging Strategies

Motivated by the previous hardness results, which point to the inherent difficulty of maximizing profit under uncertainty under both posted and discriminatory price mechanisms, we compare equivalent implementations of the heuristic nudging techniques from Section . Recalling the results that suggested that nudging by using a posted price may be a less costly mechanism than seeding to get consumers of one technology to switch to the other, we now give empirical evidence that the heuristic approaches to increasing profit under uncertainty favor a posted price mechanism, rather than seeding.[8]

[5]The results also hold for a profit maximization problem with the additional constraint of increasing market share from a given starting equilibrium, i.e. requiring that $x_2^* > x_2^o$.

[6]We refer the reader to the full paper for proof details.

[7]Altering a posted price discount mechanism so that only non-adopters are charged the new price amounts to allowing discriminatory pricing. In this case, profit maximization is again achieved under full market penetration and is easily achieved by the same greedy algorithm.

[8]Our nudging mechanisms are also applicable to welfare maximization. The full paper shows that higher quality technology adoption is better from a welfare perspective.

We set Δq and Δp so that profit is always increasing for a larger market share of technology 2 for all possible starting equilibria $x_2^o < 1$; we do this simply for clarity of the presentation. For both seeding and posted price mechanisms, we naively fix the probability that the mechanism will encourage a single additional technology 1 adopter to switch to technology 2. For seeding, using Equation 4, we choose the number of players to seed so that the probability that an additional technology 1 adopter will make the switch is $1 - z$. Nudging by posted price, we fix $1 - z$ to choose the size of the discount that should be applied to technology 2; see Equation 8. Recall that under uncertain information about types, the best that we can do for seeding is to pick the requisite k consumers at random from among the technology 1 adopters; we pay the ex ante expected payment to any of these k players to switch (see Equation 6).

We let externalities be additive, i.e $v(x) = x$ and θ's drawn uniformly over the interval $[0, 1]$, and fix the number of users N at 1000. The lefthand plot in Figure 1 illustrates that both "nudging" mechanisms are equally efficacious in fostering switching among technology 1 adopters. The plot shows the new equilibrium market share of technology 1 (x_1^*) as a function of initial market shares x_1^o for each mechanism. That the two curves are so closely matched is somewhat surprising: while the probability that an additional technology 1 adopter will switch is identical for the two mechanisms, the ex ante payment proffered in the case of seeding was the expected payment to foster switching rather than the *(worst-case)* payment and, consequently, we could not actually guarantee that any of the seeded users would switch to technology 2. We also note that $x_1^o - x_1^* \geq 0.081$ independently of x_1^o (except when x_1^o is small). Thus, the typical adoption avalanche amounts to over 8% of the user base! This illustrates the power of nudging mechanisms to generate a tipping point effect from any starting equilibrium.

While the two mechanisms are evenly matched in their ability to encourage switching, the righthand plot in Figure 1 shows that nudging using a posted price discount achieves a considerably better profit margin as compared to seeding. This is true despite the fact that the payment made in the case of seeding is roughly the minimum payment possible (to match the adoption switching avalanche achieved under posted price discounting). Together, both plots in Figure 1 demonstrate that nudging by posted price discount achieves the same market share increase as seeding at a substantially lower cost.

Equilibrium Pricing

Until this point we have taken prices p_1, p_2 either as exogenous parameters or endogenized the pricing of technology 2 while keeping the competitor's strategy fixed. We now proceed to endogenize the prices set by both sellers, assuming as before that costs are fixed and can therefore be ignored. Specifically, we formulate the problem as a two-stage game, where in stage 1 the two sellers simultaneously set the prices of their technologies, and the users proceed in stage 2 to make their myopic adoption choices, inducing a convergent best-response dynamic. We use simulations to study the outcomes of this game, and use iterative best-response as an algorithm to approximate the price equilibrium. In each iteration, we approximate best response to a fixed competing price by the best deviation sampled uniformly from the $[0, 1]$ interval.[9]

We summarize the outcomes of the endogenous pricing game in Figure 2. In the lefthand plot we observe several remarkable results. First, note that even when the difference in quality is not very high (10%), the seller of technology 2 charges a *higher price* in equilibrium *for every initial market share* of the technologies. However, there appears to be a phase transition between $\Delta q = 0.1$ and $\Delta q = 0.05$, with the price difference in the latter case crossing zero at roughly equal market share. The results suggest that a small increase in quality may go a long way to creating a competitive advantage for the superior technology. This positive result is in contrast with our earlier observation from the bound in Equation 2 that network effects can pose a significant barrier to entry of a higher quality technology.

The righthand plot of Figure 2 demonstrates that, as we might expect, market share strongly correlates with the degree of price competition in the market. Particularly, when either technology dominates the market, it can extract considerably higher rents from its installed base. This is especially true of the higher quality technology, which, even for very small quality differences, sees a substantial profit advantage over its lower-quality competitor when it enjoys market dominance.

Conclusion

We explored the effects of relative quality and price on the adoption dynamics of two incompatible technologies where buyers' preferences for either technology is subject to network externalities. We showed that the higher quality

[9]Recall from Section that computing a profit-optimal posted price is NP-Hard under uncertainty.

technology yields a robust consensus equilibrium, but that the externalities are responsible for equilibria where a technology of much lower quality retains considerable market share.

The monotonicity of best response adoption dynamics leads us to consider "nudging" interventions that may allow the higher quality technology seller to tip the market in its favor. We consider the general problem of profit maximization under discriminatory and non-discriminatory regimes, proving that both are NP-Hard under uncertainty. Comparing our simple discriminatory and non-discriminatory nudging strategies in this context, we find that the latter is more effective under uncertainty by growing market share at lower overall cost.

Finally, considering a game in which the pricing decisions of the two technologies are endogenous, we find that the higher quality technology can sustain a higher price even if the quality advantage is small, and that its owner can extract considerably higher rents when this technology enjoys a substantial market share.

In the spirit of recent work on viral marketing strategies on social networks [Hartline et al., 2008, Akhlaghpour et al., 2009], extending our domain to settings where the externalities have a true network embedding is interesting. Preliminary results in such a setting suggest that the structure of externalities can be leveraged to derive simple discriminatory nudging mechanisms (read seeding) that perform close to optimally for many classes of (externality) graphs.

References

Hessameddin Akhlaghpour, Mohammad Ghodsi, Nima Haghpanah, Hamid Mahini, Vahab S. Mirrokni, and Afshin Nikzad. Optimal iterative pricing with positive network externalities. working paper, 2009.

Emmanuelle Auriol and Michel Benaim. Standardization in decentralized economies. *American Economic Review*, 90(3):550–570, 2000.

F. M. Bass. A new product growth model for consumer durables. *Management Science*, 15(5):215–227, 1969.

L.M.B. Cabral. On the adoption of innovation with network externalities. *Mathematical Social Sciences*, 19:299–308, 1990.

Joseph Farrell and Garth Saloner. Standardization, compatibility, and innovation. *Rand Journal of Economics*, 16(1): 70–83, 1985.

L.A. Fort and J.W. Woodlock. Early prediction of market success for grocery products. *Journal of Marketing*, 25: 31–38, 1960.

Jason Hartline, Vahab S. Mirrokni, and Mukund Sundararajan. Optimal marketing strategies over social networks. In *Seventeenth International World Wide Web Conference*, pages 189–198, 2008.

Nicole Immorlica, Jon Kleinberg, Mohammad Mahdian, and Tom Wexler. The role of compatibility in the diffusion of technologies in social networks. In *ACM Conference on Electronic Commerce*, pages 75–83, 2007.

Matthew O. Jackson and Leeat Yariv. Diffusion of behavior and equilibrium properties in network games. *American Economic Review*, 97(2):92–98, 2007.

Youngmi Jin, Soumya Sen, Roch Guerin, Kartik Hosanagar, and Zhi-Li Zhang. Dynamics of competition between incumbent and emerging network technologies. In *Third International Workshop on Economics of Networked Systems*, pages 49–54, 2008.

David Kempe, Jon Kleinberg, and Éva Tardos. Maximizing the spread of influence through a social network. In *KDD '03: Proceedings of the ninth ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 137–146, New York, NY, USA, 2003. ACM. ISBN 1-58113-737-0.

V. Mahajan, E. Muller, and Y. Wind, editors. *New-Product Diffusion Models*. Springer, 2000.

Paul Milgrom and John Roberts. Rationalizability, learning, and equilibrium in games with strategic complementarities. *Econometrica*, 58(6):1255–1277, 1990.

Stephen Morris. Contagion. *Review of Economic Studies*, 67(1):57–78, January 2000.

Arun Sundararajan. Local network effects and complex network structure. *The B.E. Journal of Theoretical Economics*, 7(1):46, 2007.